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Critical Current and Electron Depairing in Superconducting Films*

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By varying the magnetic flux enclosed by a cylindrical superconducting film, a supercurrent can be induced to flow in the film. A maximum in the current is reached at a certain value of flux, beyond which the current gradually decreases as a result of the breakup of electron pairs. The self-inductance of the cylinder prevents, however, the experimental study of the depairing situation. This difficulty can be overcome by filling the hollow cylinder with another superconductor whose Meissner effect reduces the flux linkage and the self-inductance of the film. This method of stabilizing the electron depairing should make possible the confirmation of both of the following effects of pair breakup: (a) almost normal tunneling current for voltages less than the gap voltage; (b) anisotropic-electromagnetic surface impedance of the supercurrent-carrying film.

THE most basic property of a superconducting film is, of course, the possibility of a supercurrent with the total absence of resistance. It is consequently of considerable interest to understand the factors which limit the magnitude of the supercurrent and the circumstances under which it is caused to vanish. Unfortunately, as is discussed in more detail below, the two types of experimental arrangements used to date for investigating this question either produce an abrupt break in the superconducting coherence when the maximum current strength is reached or else produce no significant change, and, consequently, give little insight into the nature of the current-carrying state. Bardeen,[1] Rogers,[2] and Parmenter[3] have discussed the decrease in the current which is predicted by the BCS theory[4] as the common momentum of the electron pairs is increased past the value at which the pairs begin to be energetically unstable with respect to break up into individual quasiparticle excitations. Bardeen[1] has noted, however, that this interesting depairing regime is relatively inaccessible to experimental investigation. The purpose of the present note is to point out a possible experimental technique for exploring this regime, and to discuss some of the general features of the depairing which one can hope to verify. Although the discussion is limited to zero temperature and to the BCS model, the qualitative conclusions do not depend upon these simplifications.

In order to pass through the depairing regime, the film must not be simply connected, but rather, say, must be in the form of a hollow cylinder. The velocity of the superconducting pairs is then determined by inductive coupling with a variable magnetic field. Consequently, the following London-type equation relates the current per unit length J which flows around the axis of the cylinder to the vector potential A of the field:

$$J=-(c\tau/4\pi\lambda_L^2)A_s f((A-A_0)/A_s), \qquad (1)$$

A_0 specifies the quantum state of the superconducting pair field; (its discreteness is not essential to the present discussion and it can be regarded as a continuous variable). A_s is a characteristic value for depairing and is equal in the BCS model to $c\Delta/ev_F$, where c is the velocity of light, e the electron charge, v_F the Fermi velocity, and Δ one-half of the energy gap. λ_L is the London penetration depth and τ is an effective thickness of the film. Because of the Pippard nonlocal effect, τ can be expected to be smaller than the actual thickness. As discussed by Bardeen[1], $f(x)=x$ for $x<1$ and reaches a maximum of 1.01 at $x=1.03$. For larger values of x it decreases monotonically, reaching zero at $x=1.36$. A plot of the current characteristic as $-J$ versus A (for $A_0=0$) is shown in Fig. 1 for the BCS model.

Equation (1) does not determine directly the value of J, as the magnetic field set up by J produces large changes in A. If A_{ex} is the vector potential of an external uniform field parallel to the axis of the cylinder and L is the self-inductance of a unit length of the cylinder,

$$A=A_{ex}+(L/l)J, \qquad (2)$$

where l is the circumference of a cross section of the cylinder. Plotted in Fig. 1, this gives a straight line "load-line" plot of $-J$ vs A with the negative slope $-l/L$; the $-J=0$ intercept is $A=A_{ex}$. Equations (1) and (2) can readily be solved in the weak field linear range of $f(x)$ to give

$$A=(\gamma A_{ex}+A_0)/(1+\gamma), \qquad (3)$$

where

$$\gamma=4\pi l\lambda_L^2/c\tau L=l\lambda_L^2/\tau\Sigma. \qquad (4)$$

Σ is the cross-sectional area of the cylinder. As a simple application of this equation, imagine that initially t' cylinder is cooled below the transition temperature ' field $A_{ex}=A_0$. The cylinder enters the supercondu' state, but with no current flowing. The externa is now removed adiabatically so that A_0, being

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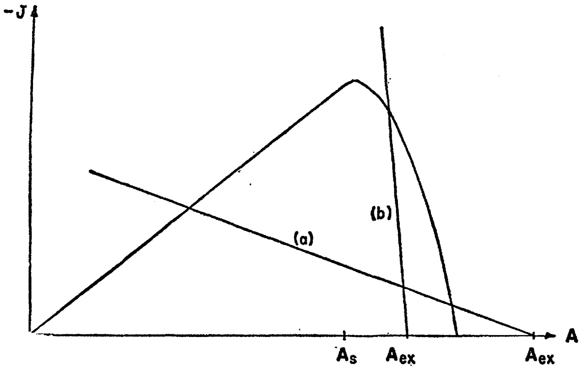

FIG. 1. Superconducting current $-J$ versus vector potential A. For $A>A_s$ (see text) depairing occurs. The resulting "broken" pairs of individual particles carry normal current which tends to cancel the supercurrent and reduce it to zero at $A=1.36\ A_s$ for the BCS model. The intersection with the load line, which represents the self-induction of the film, determines the actual A and $-J$ for an applied A_{ex}. With a flat load line (a) (high inductance) most of the depairing region is inaccessible. Only with a steep load line (b), obtainable with a "sandwich," can the depairing region be explored.

tional to the quantum number of the pair wave function, remains unchanged as A_{ex} becomes zero. Consequently, we find that the trapped flux is diminished by the ratio

$$\frac{A}{A_0}=(1+\gamma)^{-1}=(1+2\lambda_L^2/\tau r)^{-1}, \tag{5}$$

where r is the radius of the cylinder. This reduction in the magnitude of the flux quantum for small thin cylinders has been noted by Blatt,[5] Bardeen,[5] and Keller and Zumino.[6]

Solutions of the simultaneous equations (1) and (2) for stronger fields outside the linear range of $f(x)$ are obtained as the intersection of the current characteristic with the load line. The effect of increasing the external field can be visualized as a parallel translation of the load line so that its intersection with the abscissa is shifted continuously to larger values. Clearly, to explore the entire depairing regime, it is necessary that the slope of the load line be greater in magnitude than the maximum slope of the current characteristic, which is found in the BCS model to be $f'(1.36)=-6$. This gives the requirement

$$\gamma>6. \tag{6}$$

Since the factor l/Σ is equal to $2/r$ for a hollow cylinder, inequality (6) requires an impossibly small radius. This is a result of the relatively large cross-sectional area and, hence, large self-inductance of the cylinder, and can be remedied by the simple expedient of filling the interior of the cylinder by some suitable "inert" supercondcutor. The Meissner effect then excludes the bulk of the interior region from contributing to the inductance and we obtain $\Sigma=l\lambda'$, where λ' is the penetration depth for the inner solid cylinder plus the thickness of the insulating layer separating it from the outer film being studied. With this arrangement, we obtain by Eq. (4),

$$\gamma=\lambda_L^2/\tau\lambda', \tag{7}$$

independent of the radius of the cylinder. Since λ' is of the same order as λ_L, while τ can easily be made an order of magnitude smaller, there is now no difficulty in making $\gamma>6$.

With inequality (6) satisfied, there is a unique intersection of the load line with the current characteristic for all fields $A_{ex}\leq 1.36A_s$, and the entire characteristic can be traced out. As A_{ex} exceeds 1.36 A_s, quantum transitions are forced upon the pair field so that A_0 increases from zero to $A_0=A_{\mathrm{ex}}-1.36\ A_s$. Upon decreasing A_{ex}, the system follows the new characteristic shifted by A_0 with respect to the original.

Correspondingly, the current decreases continuously to zero and remains zero until A_{ex} begins to decrease, at which point the current immediately beings to increase. A natural experiment for testing this prediction would be of the type already performed by Mercereau and Crane[7] for $\gamma\ll 1$. The pickup coil would now have to be outside the cylinder—and wound as tightly around it as possible. But it should be noted that the cylindrical geometry is by no means essential. A thick flat rectangular base plate of superconductor with an insulating layer and the film to be studied on one side serves equally well. When placed in a magnetic field parallel to two of the edges a supercurrent flows along the film provided good electrical contact is established with the base plate at the two edges. Such a "sandwich" could be incorporated as one wall of a resonant microwave cavity and the effective penetration depth measured in the manner of Faber and Pippard.[8] As the dc field is increased sufficiently to bring the film into the depairing regime, both a field dependence and a marked anisotropy should be observable. The ratio of the effective penetration depth of the sandwich to λ' (neglecting film thickness) is given by dA/dA_{ex} when the fluctuating magnetic field is parallel to the dc field, and by A/A_{ex} for the perpendicular case. Note that in the parallel case the film supplies a *negative* ac London current and an *antiscreening*, thereby *lengthening* the penetration depth.

The most important aspect of the depairing phenomenon is that it provides a gradual loss of the superconducting coherence and finally a second-order phase transition. The abrupt transition encountered when a film is used as a circuit element in series with a current generator is not a true indication of the basic nature of the coherence in the superconducting state. In this case, a sudden increase in the internal energy of the film

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takes place as the coherence is broken. Inductive coupling even with $\gamma \ll 1$ is already better in that the coherence is not artificially broken by demanding more current than the superconducting film can supply. But with $\gamma \ll 1$, the film is forced to make quantum jumps to other values of A_0 at essentially the maximum current points (as established experimentally by Mercereau and Crane[7]) and most of the depairing regime is never entered. Only with the sandwiches can we achieve $\gamma > 6$ and, as a result of blocking available momentum space by the unpaired electrons, reduce the energy gap gradually to zero. For this case it is easy to see that the free energy rises monotonically as a function of A_{ex} to its normal state value at $A_{ex} = 1.36\, A_s$, and that a second-order transition does indeed take place. (Actually, the film never really enters the normal state but remains, for $A_{ex} > 1.36\, A_s$, as explained above, at the threshold between the normal state and one of the completely depaired superconducting states.) For $\gamma < 6$, the free energy rises monotonically above the normal-state value into a metastable region. Although each individual quantum state exhibits a cusp in the plot of free energy as A_{ex}, the return branch is inaccessible and the film follows the envelope of the cusps (constant free energy) as soon as a cusp is reached.

For $\gamma > 6$, tunneling from the outside film of the sandwich should provide a convenient check on the expected monotonic decrease of the energy gap in the depairing regime. The results of detailed computations of tunneling current versus voltage (I vs V) will be presented elsewhere and it suffices here to describe them qualitatively. It is important to distinguish between the energy gap parameter Δ, which is a true measure of the superconducting coherence, and the "gap" in the I vs V curve, which is of less fundamental significance. This is clear for $A = A_s$ where depairing has not yet started, but the I versus V curve has nonvanishing I for all values of V (varying as $V^{3/2}$ for $V \approx 0$). For A somewhat greater than A_s there are already enough low-energy unpaired electrons that the I versus V curve is considerably filled in at low voltages, although $\Delta \neq 0$.

Note added in proof. Because of interaction of the electrons with the film surfaces, depairing and the decrease in the gap already begin to occur at low fields as calculated by Douglass[9] from the Ginzburg-Landau theory. Consequently, the function $f(x)$ describing the $-J$ versus A current characteristic has a broader maximum than shown in Fig. 1. But the above general remarks concerning the effect of self-inductance remain unchanged. In particular, the first-order transition found by Douglass is a result only of geometry, and becomes second order for a sandwich.

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Determination of the Size of the "Necks" in Fermi Surfaces of Even-Valence Metals: Application to Lead*

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A technique is described which permits a precise measurement of features in the magnetoresistance which are directly related to the Fermi surface "neck" size of *even*-valence metals. The axis of the sample is tipped through a precisely known angle, by means of a goniometer arc, causing the disappearance of cusp-like troughs which indicate the boundaries of the particular features. The dimensions of these features are determined for Pb within uncertainties an order of magnitude smaller than those of previous measurements. The neck size for Pb, as calculated within the framework of Gold's free-electron model for the Fermi surface, is $0.29b$, where b is the period of the reciprocal lattice.

I. INTRODUCTION

THE existence of open (multiply connected) trajectories or orbits on the Fermi surfaces of metals and their directions in momentum space can be easily determined by means of high-field magnetoresistance measurements.[1] For monovalent metals, this measurement yields, within the framework of a reasonable model for the Fermi surface such as the nearly free electron model, a very precise calipering of the size of the necks (usually the intersections of the Fermi surface with the Brillouin zone boundaries).[2] With even valence metals, only a very crude estimate of the neck size can be made from data obtained with the conventional techniques. The primary purpose of this communication is to demonstrate the feasibility of a magnetoresistance technique

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